

ENTERING ALGEBRA I

Welcome to Algebra!! In preparation for the upcoming year it is important that you fully understand the following concepts from Pre-Algebra. You can expect to be challenged and you can expect that Algebra is much more complex than Pre-Algebra. Prepare to work in class as well as at home. Prepare to never give up. Prepare to give your best. Prepare to expand your thinking. ☺

You may find the following websites useful when reviewing last year's objectives:

<http://www.coolmath.com/>

<https://www.purplemath.com/>

<https://mathway.com/>

<http://www.freemathhelp.com/>

<https://www.khanacademy.org/>

<http://www.cymath.com/>

Evaluating Algebraic Expressions

1. Substitute the given values for the variables in the expression
2. Evaluate the expression using the order of operations
 - Parentheses/Brackets (inside to outside)
 - Exponents
 - Multiplication/Division (left to right)
 - Addition/Subtraction (left to right)

ex: $9x^2 - 4(y + 3z)$
for $x = -3, y = 2, z = 5$

$$9(-3)^2 - 4(2 + 3 \cdot 5)$$

$$9(-3)^2 - 4(2 + 15)$$

$$9(-3)^2 - 4 \cdot 17$$

$$9 \cdot 9 - 4 \cdot 17$$

$$81 - 4 \cdot 17$$

$$81 - 68 = \boxed{13}$$

The Distributive Property

1. Multiply the number outside the parentheses by each term in the parentheses.
2. Keep the addition/subtraction sign between each term.

ex: $5(8x - 3)$

$$5(8x - 3)$$

$$5(8x) - 5(3)$$

$$\boxed{40x - 15}$$

Simplifying Algebraic Expressions

1. Clear any parentheses using the Distributive Property
2. Add or subtract like terms (use the sign in front of each term to determine whether to add or subtract)

ex: $2(3x - 4) - 12x + 9$

$$2(3x - 4) - 12x + 9$$

$$6x - 8 - 12x + 9$$

$$\boxed{-6x + 1}$$

Evaluate each expression for $a = 9$, $b = -3$, $c = -2$, $d = 7$. Show your work.

| | | | |
|-------------------|-----------------------|--------------------------|----------------------------|
| 1. $a - cd$ | 2. $2b^3 + c^2$ | 3. $\frac{a + d - c}{b}$ | 4. $(a - b)^2 + d(a + c)$ |
| 5. $4c - (b - a)$ | 6. $\frac{a}{b} - 5a$ | 7. $2bc + d(12 - 5)$ | 8. $b + 0.5[8 - (2c + a)]$ |

Simplify each expression using the Distributive Property.

| | | | |
|----------------|----------------|------------------|------------------|
| 9. $5(2g - 8)$ | 10. $7(y + 3)$ | 11. $-3(4w - 3)$ | 12. $(6r + 3)^2$ |
|----------------|----------------|------------------|------------------|

Simplify each expression, showing all work.

| | | | |
|-------------------------------|-----------------------------|----------------------------|------------------------------|
| 13. $8(x + 1) - 12x$ | 14. $6w - 7 + 12w - 3z$ | 15. $9n - 8 + 3(2n - 11)$ | 16. $3(7x + 4y) - 2(2x + y)$ |
| 17. $(15 + 8d)(-5) - 24d + d$ | 18. $9(b - 1) - c + 3b + c$ | 19. $20f - 4(5f + 4) + 16$ | 20. $8(h - 4) - h - (h + 7)$ |

Solving One-Step Equations

1. Cancel out the number on the same side of the equal sign as the variable using inverse operations (addition/subtraction; multiplication/division)
2. Be sure to do the same thing to both sides of the equation!

ex: $-18 = 6j$

$$\frac{-18}{6} = \frac{6j}{6}$$

$$-3 = j \rightarrow \boxed{j = -3}$$

Solving Two-Step Equations

1. Undo operations one at a time with inverse operations, using the order of operations in reverse (i.e. undo addition/subtraction before multiplication/division)
2. Be sure to always do the same thing to both sides of the equation!

ex: $\frac{a}{7} - 12 = -9$

$$\frac{a}{7} - 12 = -9$$

$$+ 12 \quad + 12$$

$$\frac{a}{7} = 3 \times 7$$

$$\boxed{a = 21}$$

Solving Multi-Step Equations

1. Clear any parentheses using the Distributive Property
2. Combine like terms on each side of the equal sign
3. Get the variable terms on the same side of the equation by adding/subtracting a variable term to/from both sides of the equation to cancel it out on one side
4. The equation is now a two-step equation, so finish solving it as described above

ex: $5(2x - 1) = 3x + 4x - 1$

$$10x - 5 = 3x + 4x - 1$$

$$10x - 5 = 7x - 1$$

$$- 7x \quad - 7x$$

$$3x - 5 = -1$$

$$+ 5 \quad + 5$$

$$3x = 4$$

$$\frac{3x}{3} = \frac{4}{3}$$

$$\boxed{x = \frac{4}{3}}$$

Solve each equation, showing all work.

| | | | |
|-----------------------------|-------------------------------|------------------------------|--------------------------------|
| 21. $f - 64 = -23$ | 22. $-7 = 2d$ | 23. $\frac{b}{-12} = -6$ | 24. $13 = m + 21$ |
| 25. $5x - 3 = -28$ | 26. $\frac{w + 8}{-3} = -9$ | 27. $-8 + \frac{h}{4} = 13$ | 28. $22 = 6y + 7$ |
| 29. $8x - 4 = 3x + 1$ | 30. $-2(5d - 8) = 20$ | 31. $7r + 21 = 49r$ | 32. $-9g - 3 = -3(3g + 2)$ |
| 33. $5(3x - 2) = 5(4x + 1)$ | 34. $3d - 4 + d = 8d - (-12)$ | 35. $f - 6 = -2f + 3(f - 2)$ | 36. $-2(y - 1) = 4y - (y + 2)$ |

Before you can add or subtract fractions with different denominators, you must first find equivalent fractions with the same denominator, like this:

1. Find the smallest multiple (LCM) of both numbers in the denominator.
2. Rewrite the fractions as equivalent fractions with the LCM as the denominator.

Ex: $\frac{1}{2} + \frac{1}{3}$

Find the LCM of 2 and 3; 6. So, the Least Common Denominator is 6.

Change the $\frac{1}{2}$:

$$\frac{1 \times 3}{2 \times 3} = \frac{3}{6}$$

Change the $\frac{1}{3}$:

$$\frac{1 \times 2}{3 \times 2} = \frac{2}{6}$$

$$\text{So } \frac{1}{2} + \frac{1}{3} = \frac{3}{6} + \frac{2}{6}$$

Now you can do it!

$$\frac{3+2}{6} = \frac{5}{6}$$

*If the denominators are the same:

$$\frac{3}{8} + \frac{4}{8} = \frac{3+4}{8} = \frac{7}{8}$$

Negative Exponents & Simplifying Monomials

Zero Exponent: Any number raised to the zero power equals 1

ex: $y^0 = 1$

Negative Exponent: Move the base to the opposite side of the fraction line and make the exponent positive

ex: $x^{-4} = \frac{1}{x^4}$

Monomial x Monomial: Multiply the coefficients and add the exponents of like bases

ex: $(4x^3)(2x^5) = 8x^8$

Monomial ÷ Monomial: Divide the coefficients and subtract the exponents of like bases

ex: $\frac{a}{a^6} = a^{-5} = \frac{1}{a^5}$

Power of a Monomial: Raise each base (including the coefficient) to that power. If a base already has an exponent, multiply the two exponents

ex: $(-2fg^5)^3 = -8f^3g^{15}$

Power of a Quotient: Raise each base (including the coefficient) to that power. If a base already has an exponent, multiply the two exponents

ex: $\left(\frac{5d^3}{c}\right)^2 = \frac{25d^6}{c^2}$

| | | | |
|---------------------------------|----------------------------------|----------------------------------|-----------------------------------|
| 37. $\frac{8}{2} + \frac{3}{3}$ | 38. $\frac{1}{9} - \frac{2}{15}$ | 39. $\frac{17}{2} + \frac{3}{5}$ | 40. $\frac{23}{2} + \frac{9}{4}$ |
| 41. $\frac{8}{3} + \frac{3}{2}$ | 42. $\frac{1}{6} + \frac{2}{3}$ | 43. $\frac{9}{7} - \frac{5}{6}$ | 44. $\frac{5}{2} + \frac{13}{12}$ |

Simplify each expression. Write your answers using only positive exponents.

| | | | |
|-------------------------------|---|---|--|
| 45. w^{-9} | 46. $\frac{m^5}{m^2}$ | 47. $f^5 \cdot f^3$ | 48. $\left(\frac{h^2}{g}\right)^3$ |
| 49. $(a^5)^2$ | 50. $\frac{1}{b^{-3}}$ | 51. z^0 | 52. $4r^6 \cdot 3r \cdot 2r^2$ |
| 53. $\frac{9p^{-2}}{3q^{-3}}$ | 54. $\frac{8d^3}{2cd^{-2}}$ | 55. $(g^4h)^2 \cdot (2g^3h^{-1})^2$ | 56. $(6a)^0$ |
| 57. $(-3n^2k^4)^2$ | 58. $\left(\frac{w^5x^{-2}y}{w^2xy^4}\right)^3$ | 59. $\frac{6 \cdot 10^7}{2 \cdot 10^3}$ | 60. $(1.5 \cdot 10^{-6}) \cdot (4 \cdot 10^9)$ |

Slope & Rate of Change

Finding the Slope Given Two Points: Use the coordinates from the points in the slope formula:

$$\text{Slope } (m) = \frac{y_2 - y_1}{x_2 - x_1}$$

ex: $(4, -2), (-3, 8)$
 $x_1 \quad y_1 \quad x_2 \quad y_2$

$$m = \frac{8 - (-2)}{-3 - 4} = \frac{10}{-7} = \boxed{-\frac{10}{7}}$$

Finding the Rate of Change From a Table: Determine the amount the dependent variable (y) is changing and the amount the independent variable (x) is changing.

$$\text{Rate of Change} = \frac{\text{change in } y}{\text{change in } x}$$

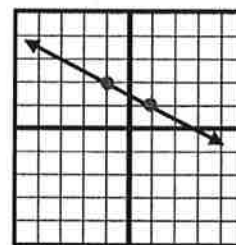
ex:

| | | | | | |
|---|-----------|----|-----|-----|-----|
| x | # months | 3 | 5 | 7 | 9 |
| y | Cost (\$) | 80 | 130 | 180 | 230 |

$\overset{+2}{\curvearrowright}$ $\overset{+2}{\curvearrowright}$ $\overset{+2}{\curvearrowright}$
 $\underset{+50}{\curvearrowleft}$ $\underset{+50}{\curvearrowleft}$ $\underset{+50}{\curvearrowleft}$

$$m = \frac{50}{2} = \boxed{25 \text{ dollars/month}}$$

Finding the Slope From a Graph: Choose 2 points on the graph. Find the vertical change (rise) and horizontal change (run) between the 2 points and write it as a fraction $\frac{\text{rise}}{\text{run}}$. (Up is positive, down is negative, right is positive, and left is negative).



rise = +1
run = -2

$$m = \frac{1}{-2} = \boxed{-\frac{1}{2}}$$

Graphing Linear Equations

Slope-Intercept Form: $y = mx + b$
 ↑ ↑
 slope y-intercept

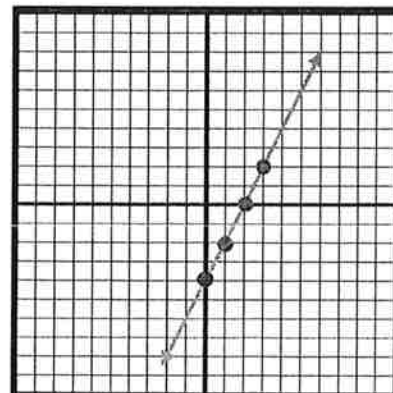
ex: $y = 2x - 4$

y-intercept: -4

slope: $2 = \frac{2}{1}$ ← rise
← run

How To Graph:

1. Make a point on the y-axis at the y-intercept.
2. Use the slope to determine where to make the next point. The numerator tells you the rise (how far up/down) and the denominator tells you the run (how far right/left) to make the next point.
3. Repeat to make more points and then connect the points with a line.



61. $(-5, 3), (2, 1)$

62. $(8, 4), (11, 6)$

63. $(9, 3), (9, -1)$

64. $(-4, -2), (-6, 4)$

Find the rate of change. Show your work.

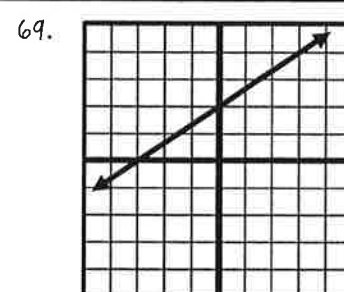
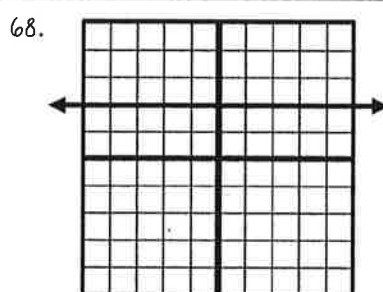
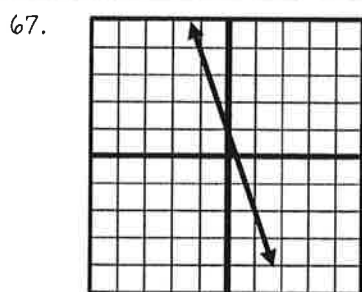
65.

| | | | | |
|---------------------|-----|-----|-----|-----|
| Number of Hours | 3 | 6 | 9 | 12 |
| Distance (in miles) | 135 | 270 | 405 | 540 |

66.

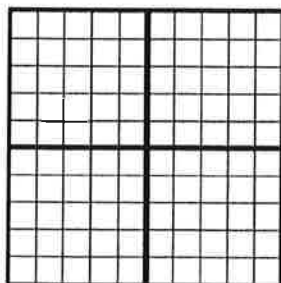
| | | | | |
|-----------------|-----|-----|-----|-----|
| Number of Weeks | 1 | 3 | 5 | 7 |
| Pounds | 173 | 169 | 165 | 161 |

Find the slope of the line.

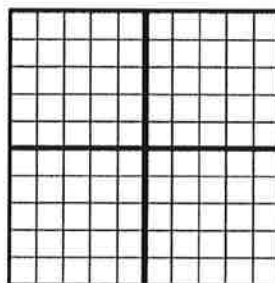


Graph the line.

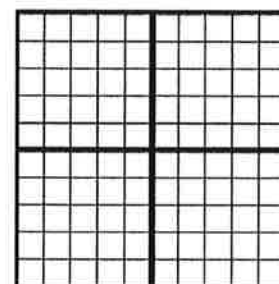
70. $y = -x - 3$



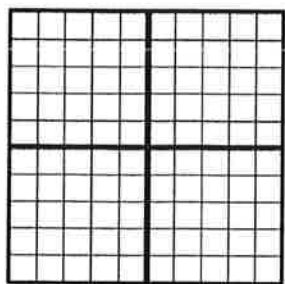
71. $y = \frac{1}{3}x + 2$



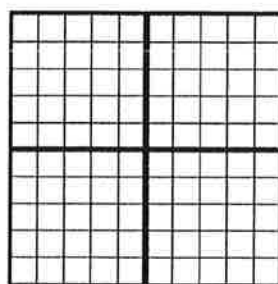
72. $y = -3x - 1$



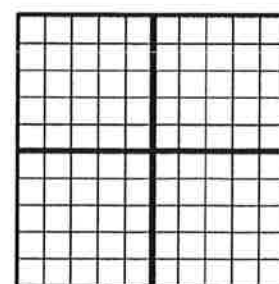
73. $y = -\frac{3}{2}x - 2$



74. $y = 2x + 1$



75. $y = \frac{1}{4}x$



Solving Proportions

1. Set the two cross-products equal to each other
2. Solve the equation for the variable

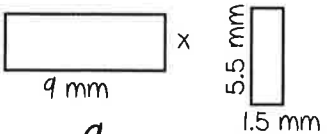
ex: $\frac{m}{4} = \frac{3}{5}$

$$\frac{5m}{5} = \frac{12}{5}$$

$$m = 2.4$$

Similar Figures

1. To find a missing side length, set up a proportion, matching up corresponding sides.
2. Solve the proportion using the steps above.

ex: 

$$\frac{x}{1.5} = \frac{9}{5.5}$$

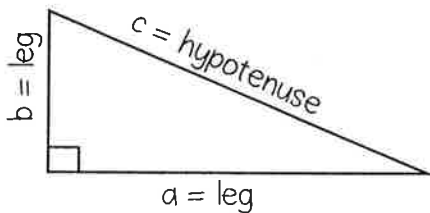
$$x = 2.45 \text{ mm}$$

The Pythagorean Theorem

*** The Pythagorean Theorem applies to right triangles only **

The sides next to the right angle (a & b) are legs

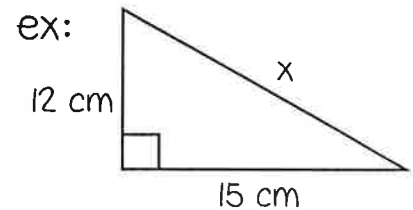
The side across from the right angle (c) is the hypotenuse



Pythagorean Theorem: $a^2 + b^2 = c^2$

To find the hypotenuse: add the squares of the legs and then find the square root of the sum

To find a leg: subtract the square of the given leg from the square of the hypotenuse and then find the square root of the difference



x is the hypotenuse

$$12^2 + 15^2 = x^2$$

$$144 + 225 = x^2$$

$$369 = x^2$$

$$x = \sqrt{369} \approx 19.2 \text{ cm}$$

ex: $a = ?$, $b = 3$, $c = 6$

a is a leg

$$a^2 + 3^2 = 6^2$$

$$a^2 + 9 = 36$$

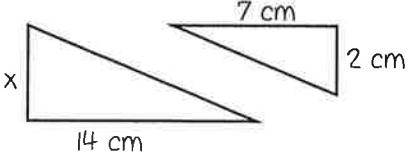
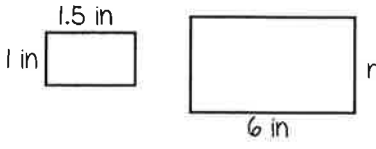
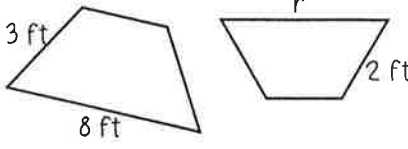
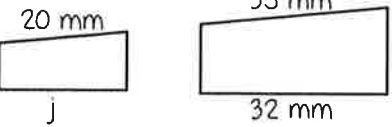
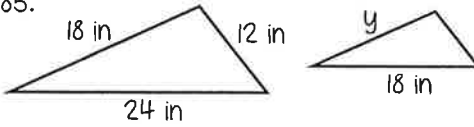
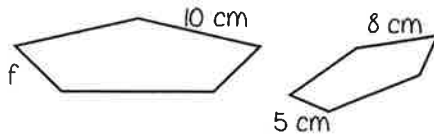
$$a^2 = 36 - 9 = 27$$

$$a = \sqrt{27} \approx 5.2$$

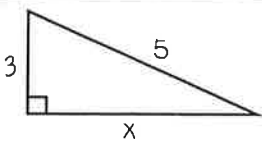
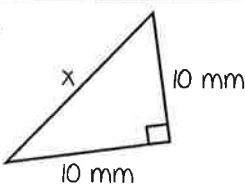
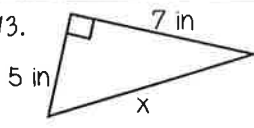
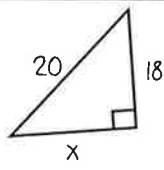
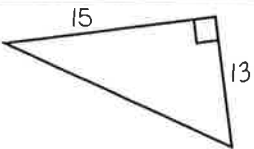
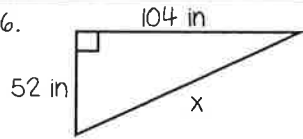
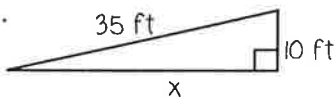
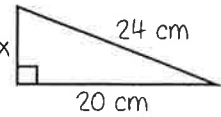
Solve each proportion, showing all work.

| | | | | |
|---------------------------------|----------------------------------|---------------------------------|-----------------------------------|----------------------------------|
| 76. $\frac{6}{7} = \frac{4}{m}$ | 77. $\frac{12}{5} = \frac{k}{3}$ | 78. $\frac{h}{7} = \frac{8}{2}$ | 79. $\frac{22}{n} = \frac{9}{36}$ | 80. $\frac{4}{2i} = \frac{3}{c}$ |
|---------------------------------|----------------------------------|---------------------------------|-----------------------------------|----------------------------------|

Assume each pair of figures is similar. Find the missing side length, showing all work.

| | | |
|---|--|---|
| 81.  | 82.  | 83.  |
| 84.  | 85.  | 86.  |

Find the missing side length in each right triangle to the nearest tenth. Show your work!

| | | | |
|---|---|--|---|
| 87. $a = 6, b = 8, c = ?$ | 88. $a = ?, b = 9 \text{ cm}, c = 13 \text{ cm}$ | 89. $a = 7, b = ?, c = 14$ | 90. $a = 14 \text{ in}, b = 14 \text{ in}, c = ?$ |
| 91.  | 92.  | 93.  | 94.  |
| 95.  | 96.  | 97.  | 98.  |

Determine whether or not you can form a right triangle from the given side lengths. Explain.

| | |
|----------------|----------------|
| 99. 18, 22, 26 | 100. 5, 12, 13 |
|----------------|----------------|