

Pre-Calculus Review

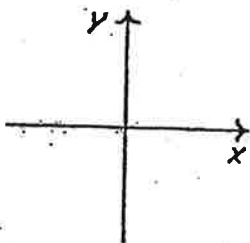
"And we're off . . . ☺"

Graphical Analysis

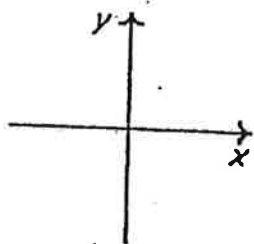
Chapter 1 deals with functions and their characteristics. To facilitate a study of functions, it is important to visualize mentally the graphical image of a function when given an algebraic description.

I. Graph each function. Clearly indicate units on the axes provided.

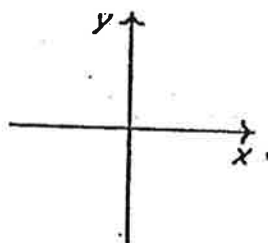
1. $f(x) = x$



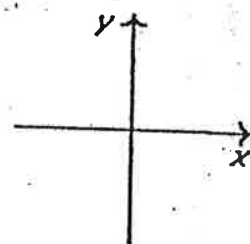
2. $f(x) = x^2$



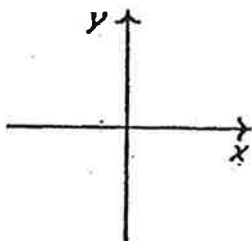
3. $f(x) = x^3$



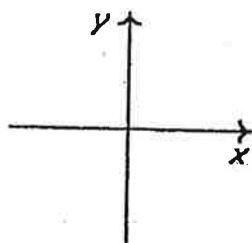
4. $f(x) = |x|$



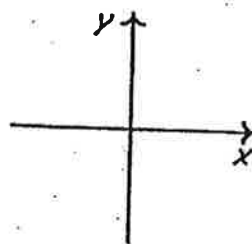
5. $f(x) = [x]$



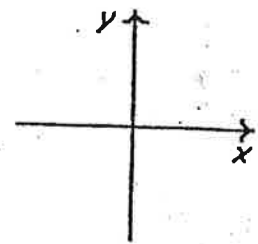
6. $f(x) = \sin x$



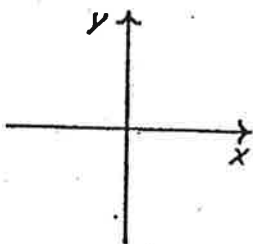
7. $f(x) = \cos x$



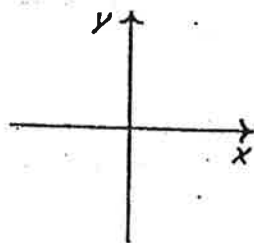
8. $f(x) = \tan x$



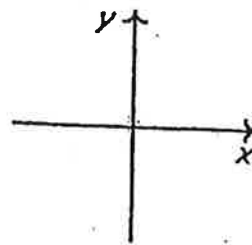
9. $f(x) = \sec x$



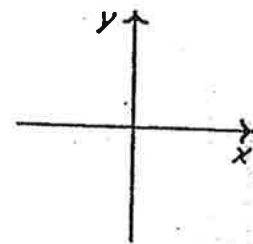
10. $f(x) = 2^x$



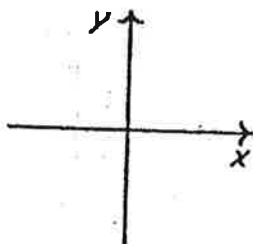
11. $f(x) = \log_2 x$



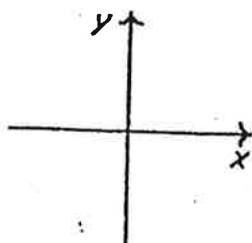
12. $f(x) = \frac{1}{x}$



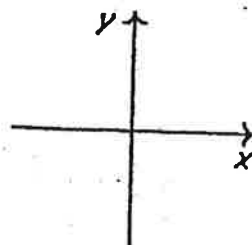
13. $f(x) = \frac{1}{x^2}$



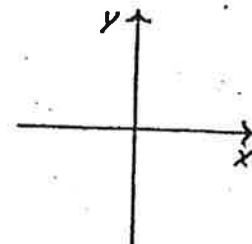
14. $f(x) = \sqrt{x}$



15. $f(x) = \sqrt{a^2 - x^2}$



16. $f(x) = \begin{cases} 0, & \text{if } x \text{ is rational} \\ 1, & \text{if } x \text{ is irrational} \end{cases}$



This packet is a review of the entering objectives for AP Calculus.

* It is to be done NEATLY and on a SEPARATE sheet of paper. Have a great summer! *



I. Simplify. Show the work that leads to your answer.

1. $\frac{x-4}{x^2-3x-4}$

2. $\frac{x^3-8}{x-2}$

3. $\frac{5-x}{x^2-25}$

4. $\frac{x^2-4x-32}{x^2-16}$

II. Trigonometric Identities.

1. Pythagorean = _____

2. $\cos 2x =$ _____

3. $\sin 2x =$ _____

III. Simplify each expression.

1. $\frac{1}{x+h} - \frac{1}{x}$

2. $\frac{2}{\frac{x^2}{10}} \cdot \frac{1}{x^5}$

3. $\frac{\frac{1}{3+x} - \frac{1}{3}}{x}$

4. $\frac{2x}{x^2-6x+9} - \frac{1}{x+1} - \frac{8}{x^2-2x-3}$

IV. Solve for z:

1. $4x + 10yz = 0$

2. $y^2 + 3yz - 8z - 4x = 0$

V. If: $f(x) = \{(3,5), (2,4), (1,7)\}$ $g(x) = \sqrt{x-3}$ $h(x) = \{(3,2), (4,3), (1,6)\}$ $k(x) = x^2 + 5$
determine each of the following:

1. $(f+h)(1) =$

2. $(k-g)(5) =$

3. $(f \circ h)(3) =$

4. $(g \circ k)(7) =$

5. $f^{-1}(x) =$

6. $k^{-1}(x) =$

7. $\frac{1}{f(x)} =$

8. $(kg)(x) =$

VI. Miscellaneous: Follow the directions for each problem.

1. Evaluate $\frac{f(x+h) - f(x)}{h}$ and simplify if $f(x) = x^2 - 2x$.

2. Expand $(x+y)^3$

3. Simplify: $x^{\frac{3}{2}}(x + x^{\frac{5}{2}} - x^2)$

VII. Expand and simplify

1. $\sum_{n=0}^4 \frac{n^2}{2}$

2. $\sum_{n=1}^3 \frac{1}{n^3}$

VIII. Simplify

1. $\frac{\sqrt{x}}{x}$

2. $e^{\ln 3}$

3. $e^{(1+\ln x)}$

4. $\ln 1$

5. $\ln e^7$

6. $\log_3(1/3)$

7. $\log_{1/2} 8$

8. $\ln \frac{1}{2}$

9. $e^{3 \ln x}$

10. $\frac{4xy^{-2}}{12x^{\frac{1}{3}}y^{-5}}$

11. $27^{2/3}$

12. $(5a^{2/3})(4a^{3/2})$

13. $(4a^{5/3})^{3/2}$

14. $\frac{3(n+1)!}{5n!}$

IX. Using the point-slope form $y - y_1 = m(x - x_1)$, write an equation for the line

1. with slope -2 , containing the point $(3, 4)$

1. _____

2. containing the points $(1, -3)$ and $(-5, 2)$

2. _____

3. with slope 0 , containing the point $(4, 2)$

3. _____

4. parallel to $2x - 3y = 7$ and passes through $(5, 1)$

4. _____

5. perpendicular to the line in problem #1, containing the point $(3, 4)$

5. _____

X. Given the vectors $\mathbf{v} = -2\mathbf{i} + 5\mathbf{j}$ and $\mathbf{w} = 3\mathbf{i} + 4\mathbf{j}$, determine

1. $\frac{1}{2}\mathbf{v}$

2. $\mathbf{w} - \mathbf{v}$

3. length of \mathbf{w}

4. the unit vector for \mathbf{v}

XI. Without a calculator, determine the exact value of each expression.

1. $\sin 0$

2. $\sin \frac{\pi}{2}$

3. $\sin \frac{3\pi}{4}$

4. $\cos \pi$

5. $\cos \frac{7\pi}{6}$

6. $\cos \frac{\pi}{3}$

7. $\tan \frac{7\pi}{4}$

8. $\tan \frac{\pi}{6}$

9. $\tan \frac{2\pi}{3}$

10. $\tan \frac{\pi}{2}$

11. $\cos(\sin^{-1} \frac{1}{2})$

12. $\sin^{-1}(\sin \frac{7\pi}{6})$

XII. For each function, determine its domain and range.

1. $y = \sqrt{x-4}$

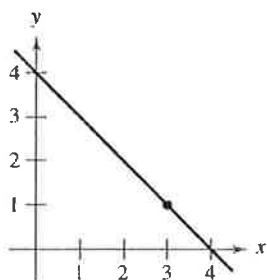
2. $y = \sqrt{x^2-4}$

3. $y = \sqrt{4-x^2}$

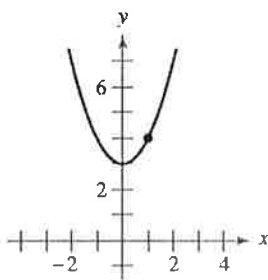
4. $y = \sqrt{x^2+4}$

In Exercises 15–24, use the graph to find the limit (if it exists).
If the limit does not exist, explain why.

15. $\lim_{x \rightarrow 3} (4 - x)$

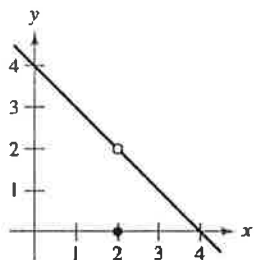


16. $\lim_{x \rightarrow 1} (x^2 + 3)$



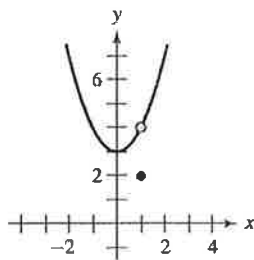
17. $\lim_{x \rightarrow 2} f(x)$

$$f(x) = \begin{cases} 4 - x, & x \neq 2 \\ 0, & x = 2 \end{cases}$$

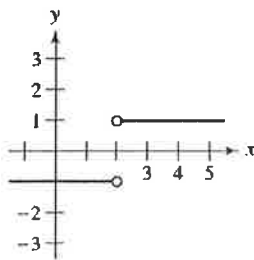


18. $\lim_{x \rightarrow 1} f(x)$

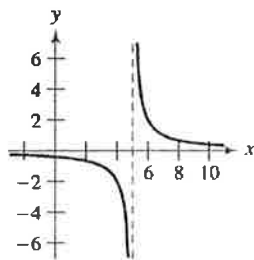
$$f(x) = \begin{cases} x^2 + 3, & x \neq 1 \\ 2, & x = 1 \end{cases}$$



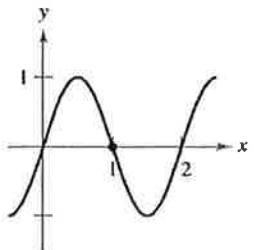
19. $\lim_{x \rightarrow 2} \frac{|x - 2|}{x - 2}$



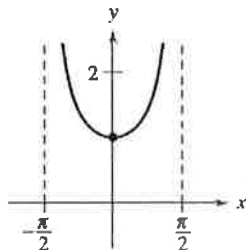
20. $\lim_{x \rightarrow 5} \frac{2}{x - 5}$



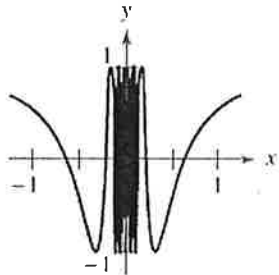
21. $\lim_{x \rightarrow 1} \sin \pi x$



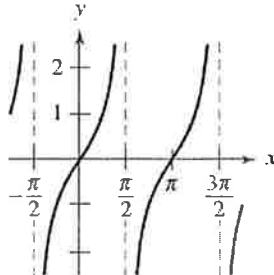
22. $\lim_{x \rightarrow 0} \sec x$



23. $\lim_{x \rightarrow 0} \cos \frac{1}{x}$



24. $\lim_{x \rightarrow \pi/2} \tan x$



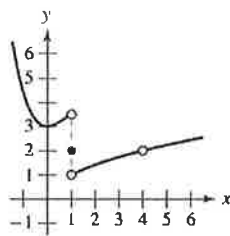
In Exercises 25 and 26, use the graph of the function f to decide whether the value of the given quantity exists. If it does, find it. If not, explain why.

25. (a) $f(1)$

(b) $\lim_{x \rightarrow 1} f(x)$

(c) $f(4)$

(d) $\lim_{x \rightarrow 4} f(x)$



26. (a) $f(-2)$

(b) $\lim_{x \rightarrow -2} f(x)$

(c) $f(0)$

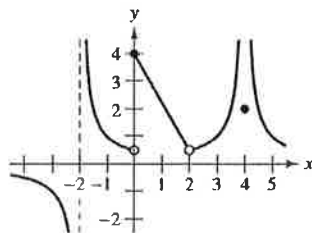
(d) $\lim_{x \rightarrow 0} f(x)$

(e) $f(2)$

(f) $\lim_{x \rightarrow 2} f(x)$

(g) $f(4)$

(h) $\lim_{x \rightarrow 4} f(x)$



Section 1.3 - Evaluating Limits Analytically:

In Exercises 5–22, find the limit.

5. $\lim_{x \rightarrow 2} x^3$

6. $\lim_{x \rightarrow -2} x^4$

15. $\lim_{x \rightarrow -4} (x + 3)^2$

16. $\lim_{x \rightarrow 0} (2x - 1)^3$

17. $\lim_{x \rightarrow 2} \frac{1}{x}$

18. $\lim_{x \rightarrow -3} \frac{2}{x + 2}$

XIII. Determine all points of intersection.

1. parabola $y = x^2 + 3x - 4$ and
line $y = 5x + 11$

2. $y = \cos x$ and $y = \sin x$ in the
first quadrant

XIV. Solve for x , where x is a real number. Show the work that leads to your solution.

1. $x^2 + 3x - 4 = 14$

2. $\frac{x^4 - 1}{x^3} = 0$

3. $(x - 5)^2 = 9$

4. $2x^2 + 5x = 8$

5. $(x + 3)(x - 3) > 0$

6. $x^2 - 2x - 15 \leq 0$

7. $12x^2 = 3x$

8. $\sin 2x = \sin x$, $0 \leq x \leq 2\pi$

9. $|x - 3| < 7$

10. $(x + 1)^2(x - 2) + (x + 1)(x - 2)^2 = 0$

11. $27^{2x} = 9^{x-3}$

12. $\log x + \log(x - 3) = 1$

XIV. Graph each function. Give its domain and range.

4. $y = x^3 - 2x^2 - 3x$

5. $y = x^2 - 6x + 1$

6. $y = \frac{x + 4}{x - 1}$

7. $y = \frac{x^2 - 4}{x + 2}$

8. $y = e^x$

10. $y = \sqrt[3]{x}$

11. $y = \ln x$

12. $y = |x + 3| - 2$

14. $y = \begin{cases} x^2 & \text{if } x < 0 \\ x + 2 & \text{if } 0 \leq x \leq 3 \\ 4 & \text{if } x > 3 \end{cases}$

In Exercises 27–30, evaluate the function as indicated.
Determine its domain and range.

27. $f(x) = \begin{cases} 2x + 1, & x < 0 \\ 2x + 2, & x \geq 0 \end{cases}$

(a) $f(-1)$ (b) $f(0)$ (c) $f(2)$ (d) $f(t^2 + 1)$

28. $f(x) = \begin{cases} x^2 + 2, & x \leq 1 \\ 2x^2 + 2, & x > 1 \end{cases}$

(a) $f(-2)$ (b) $f(0)$ (c) $f(1)$ (d) $f(s^2 + 2)$

29. $f(x) = \begin{cases} |x| + 1, & x < 1 \\ -x + 1, & x \geq 1 \end{cases}$

(a) $f(-3)$ (b) $f(1)$ (c) $f(3)$ (d) $f(b^2 + 1)$

30. $f(x) = \begin{cases} \sqrt{x + 4}, & x \leq 5 \\ (x - 5)^2, & x > 5 \end{cases}$

(a) $f(-3)$ (b) $f(0)$ (c) $f(5)$ (d) $f(10)$

In Exercises 23–26, find the limits.

23. $f(x) = 5 - x$, $g(x) = x^3$

(a) $\lim_{x \rightarrow 1} f(x)$ (b) $\lim_{x \rightarrow 4} g(x)$ (c) $\lim_{x \rightarrow 1} g(f(x))$

24. $f(x) = x + 7$, $g(x) = x^2$

(a) $\lim_{x \rightarrow -3} f(x)$ (b) $\lim_{x \rightarrow 4} g(x)$ (c) $\lim_{x \rightarrow -3} g(f(x))$

25. $f(x) = 4 - x^2$, $g(x) = \sqrt{x + 1}$

(a) $\lim_{x \rightarrow 1} f(x)$ (b) $\lim_{x \rightarrow 3} g(x)$ (c) $\lim_{x \rightarrow 1} g(f(x))$

26. $f(x) = 2x^2 - 3x + 1$, $g(x) = \sqrt[3]{x + 6}$

(a) $\lim_{x \rightarrow 4} f(x)$ (b) $\lim_{x \rightarrow 21} g(x)$ (c) $\lim_{x \rightarrow 4} g(f(x))$

In Exercises 27–36, find the limit of the trigonometric function.

27. $\lim_{x \rightarrow \pi/2} \sin x$

28. $\lim_{x \rightarrow \pi} \tan x$

29. $\lim_{x \rightarrow 1} \cos \frac{\pi x}{3}$

30. $\lim_{x \rightarrow 2} \sin \frac{\pi x}{2}$

31. $\lim_{x \rightarrow 0} \sec 2x$

32. $\lim_{x \rightarrow \pi} \cos 3x$

33. $\lim_{x \rightarrow 5\pi/6} \sin x$

34. $\lim_{x \rightarrow 5\pi/3} \cos x$

35. $\lim_{x \rightarrow 3} \tan\left(\frac{\pi x}{4}\right)$

36. $\lim_{x \rightarrow 7} \sec\left(\frac{\pi x}{6}\right)$