June 1, 2018

To My NEW AP Calculus Student,

I hope I find you enjoying your summer and anticipating the beginning of your senior year at St. Patrick Catholic High School. AP Calculus is a course designed to prepare you for the Advanced Placement test taken in May, college level Calculus I & II, as well as a higher level of thinking that will benefit you in the continuation of your educational career. I know that you are all students who plan to CONSISTENTLY work very hard throughout the year to achieve success. Part of that success begins today.

- Deadline – AP summer assignments must be submitted by 11:59 p.m. on Tuesday, July 31, 2018. Students who do not submit their AP summer assignments by the deadline will be dropped from the course.

- All AP summer assignments will be submitted online via the school website.

The material is all Algebra 3 review. I know that you all have the background you need to complete this work, but, if it is necessary for you, you may seek outside help through online resources and/or your notebooks from last year. This material is designed to remind you of previously learned concepts and/or to get you back up to speed for where you need to be at the beginning of this course. We will spend one day on the first chapter, the P-Chapter. So I insist that your best efforts be put forth in this assignment and all others throughout the school year.

This assignment will be graded, but more importantly, your understanding and thoroughness with this material is vital to the beginning of your AP Calculus journey.

If you have any questions email me at stavel@stpatrickhighschool.net. I can't wait to be a part of your senior year as we go through the Calculus journey together.

This Class IS GOING TO ROCK IT!!!

Susanne Tavel

AP Calculus Instructor

Pre-Calculus Review

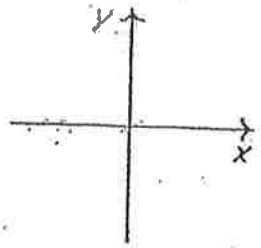
"And we're off . . . "

Graphical Analysis

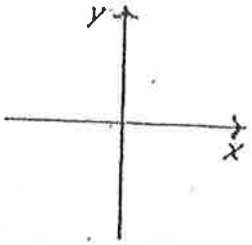
Chapter 1 deals with functions and their characteristics. To facilitate a study of functions, it is important to visualize mentally the graphical image of a function when given an algebraic description.

I. Graph each function. Clearly indicate units on the axes provided.

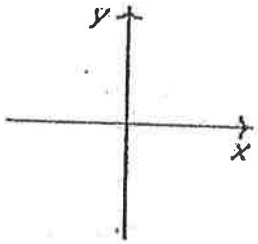
1. $f(x) = x$



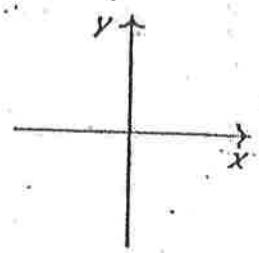
2. $f(x) = x^2$



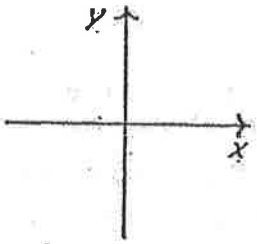
3. $f(x) = x^3$



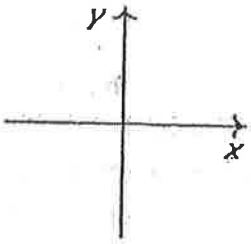
4. $f(x) = |x|$



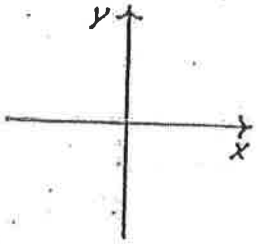
5. $f(x) = [x]$



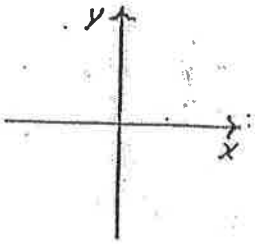
6. $f(x) = \sin x$



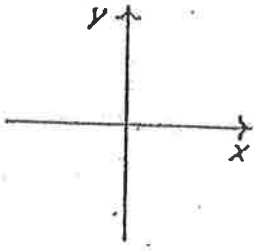
7. $f(x) = \cos x$



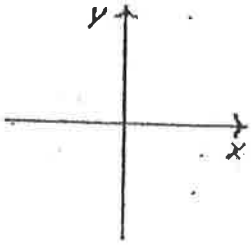
8. $f(x) = \tan x$



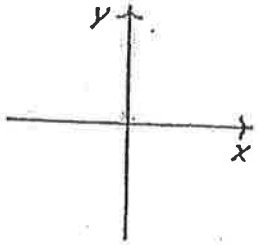
9. $f(x) = \sec x$



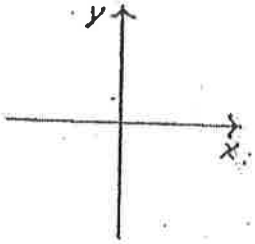
10. $f(x) = 2^x$



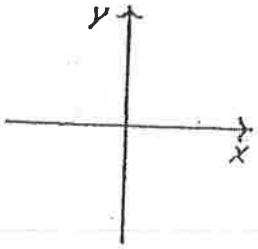
11. $f(x) = \log_2 x$



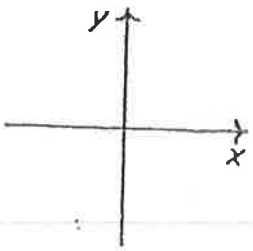
12. $f(x) = \frac{1}{x}$



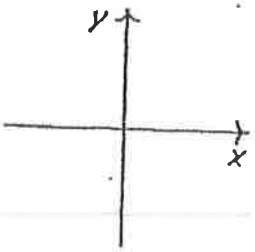
13. $f(x) = \frac{1}{x^2}$



14. $f(x) = \sqrt{x}$



15. $f(x) = \sqrt{a^2 - x^2}$



II. Answer the following questions about the indicated functions. In completing the table below use the following abbreviations, R: the set of real numbers, J: the set of integers, and N: the set of natural numbers. NOTE: This exercise may need to be done as appropriate sections of Chapter 1 are completed.

Function	Domain	Range $y = f(x)$	Roots (Find x when $f(x) = 0$)	Symmetry with respect to y-axis or origin	Even or Odd Function— $f(-x) = f(x)$ or $f(-x) = -f(x)$	Is the function periodic? If so, state the period.	Is $f(x)$ a one-to-one mapping? (For each $f(x)$ only one x exists)	State the x coordinates of any points of discontinuity
1. $f(x) = x$								
2. $f(x) = x^2$								
3. $f(x) = x^3$								
4. $f(x) = \lvert x \rvert$								
5. $f(x) = [x]$								
6. $f(x) = \sin x$								
7. $f(x) = \cos x$								
8. $f(x) = \tan x$								
9. $f(x) = \sec x$								
10. $f(x) = 2^x$								
11. $f(x) = \log_2 x$								
12. $f(x) = 1/x$								
13. $f(x) = 1/x^2$								
14. $f(x) = \sqrt{x}$								
15. $f(x) = \sqrt{a^2 - x^2}$								

This packet is a review of the entering objectives for AP Calculus.

* It is to be done NEATLY and on a SEPARATE sheet of paper. Have a great summer! *

I. Simplify. Show the work that leads to your answer.

1. $\frac{x-4}{x^2-3x-4}$ 2. $\frac{x^3-8}{x-2}$ 3. $\frac{5-x}{x^2-25}$ 4. $\frac{x^2-4x-32}{x^2-16}$

II. Trigonometric Identities.

1. Pythagorean = _____

2. cos 2x = _____

3. sin 2x = _____

III. Simplify each expression.

1. $\frac{1}{x+h}-\frac{1}{x}$ 2. $\frac{\frac{2}{x^2}}{\frac{10}{x^5}}$ 3. $\frac{\frac{1}{3+x}-\frac{1}{3}}{x}$ 4. $\frac{2x}{x^2-6x+9}-\frac{1}{x+1}-\frac{8}{x^2-2x-3}$

IV. Solve for z:

1. $4x + 10yz = 0$

2. $y^2 + 3yz - 8z - 4x = 0$

V. If: $f(x) = \{(3,5), (2,4), (1,7)\}$ $g(x) = \sqrt{x-3}$ $h(x) = \{(3,2), (4,3), (1,6)\}$ $k(x) = x^2 + 5$ determine each of the following:

1. $(f + h)(1) =$ 2. $(k - g)(5) =$ 3. $(f \circ h)(3) =$ 4. $(g \circ k)(7) =$

5. $f^{-1}(x) =$ 6. $k^{-1}(x) =$ 7. $\frac{1}{f(x)} =$ 8. $(kg)(x) =$

VI. Miscellaneous: Follow the directions for each problem.

1. Evaluate $\frac{f(x+h)-f(x)}{h}$ and simplify if $f(x) = x^2 - 2x$.

2. Expand $(x + y)^3$

3. Simplify: $x^{\frac{3}{2}}\left(x + x^{\frac{5}{2}} - x^2\right)$

VII. Expand and simplify

1. $\sum_{n=0}^{4} \frac{n^2}{2}$

2. $\sum_{n=1}^{3} \frac{1}{n^3}$

VIII. Simplify

1. $\frac{\sqrt{x}}{x}$

2. $e^{\ln 3}$

3. $e^{(1+\ln x)}$

4. ln 1

5. $\ln e^7$

6. $\log_3(1/3)$

7. $\log_{1/2} 8$

8. $\ln \frac{1}{2}$

9. $e^{3 \ln x}$

10. $\frac{4xy^{-2}}{12x^{-\frac{1}{3}}y^{-5}}$

11. $27^{2/3}$

12. $(5a^{2/3})(4a^{3/2})$

13. $(4a^{5/3})^{3/2}$

14. $\frac{3(n+1)!}{5n!}$

IX. Using the point-slope form $y - y_1 = m(x - x_1)$, write an equation for the line

1. with slope –2, containing the point (3, 4) 1. _____
2. containing the points (1, -3) and (-5, 2) 2. _____
3. with slope 0, containing the point (4, 2) 3. _____
4. parallel to $2x - 3y = 7$ and passes through (5 , 1) 4. _____
5. perpendicular to the line in problem #1, containing the point (3, 4) 5. _____

X. Given the vectors $\mathbf{v} = -2\mathbf{i} + 5\mathbf{j}$ and $\mathbf{w} = 3\mathbf{i} + 4\mathbf{j}$, determine

1. $\frac{1}{2}\mathbf{v}$
2. $\mathbf{w} - \mathbf{v}$
3. length of w
4. the unit vector for v

XI. Without a calculator, determine the exact value of each expression.

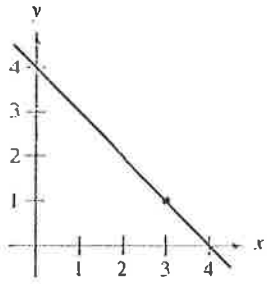
1. sin 0
2. $\sin \frac{\pi}{2}$
3. $\sin \frac{3\pi}{4}$
4. $\cos \pi$
5. $\cos \frac{7\pi}{6}$
6. $\cos \frac{\pi}{3}$
7. $\tan \frac{7\pi}{4}$
8. $\tan \frac{\pi}{6}$
9. $\tan \frac{2\pi}{3}$
10. $\tan \frac{\pi}{2}$
11. $\cos(\text{Sin}^{-1}\frac{1}{2})$
12. $\text{Sin}^{-1}(\sin \frac{7\pi}{6})$

XII. For each function, determine its domain and range.

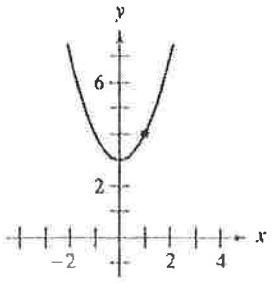
1. $y = \sqrt{x-4}$
2. $y = \sqrt{x^2-4}$
3. $y = \sqrt{4-x^2}$
4. $y = \sqrt{x^2+4}$

In Exercises 15–24, use the graph to find the limit (if it exists). If the limit does not exist, explain why.

15. $\lim_{x \to 3} (4 - x)$

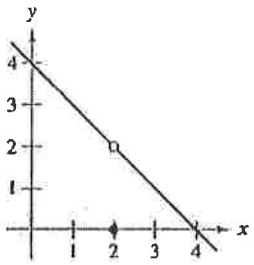


16. $\lim_{x \to 1} (x^2 + 3)$



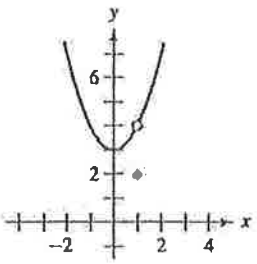
17. $\lim_{x \to 2} f(x)$

$$f(x) = \begin{cases} 4 - x, & x \neq 2 \\ 0, & x = 2 \end{cases}$$

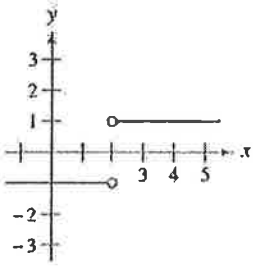


18. $\lim_{x \to 1} f(x)$

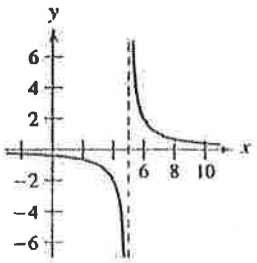
$$f(x) = \begin{cases} x^2 + 3, & x \neq 1 \\ 2, & x = 1 \end{cases}$$



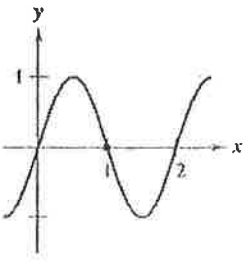
19. $\lim_{x \to 2} \dfrac{|x - 2|}{x - 2}$



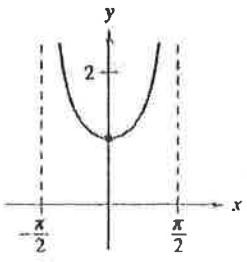
20. $\lim_{x \to 5} \dfrac{2}{x - 5}$



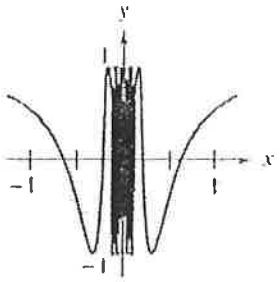
21. $\lim_{x \to 1} \sin \pi x$



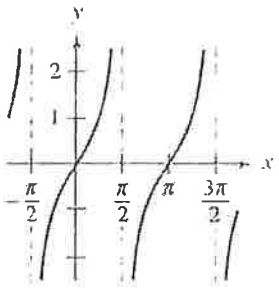
22. $\lim_{x \to 0} \sec x$



23. $\lim_{x \to 0} \cos \frac{1}{x}$

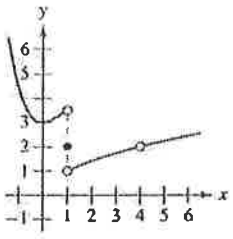


24. $\lim_{x \to \pi/2} \tan x$

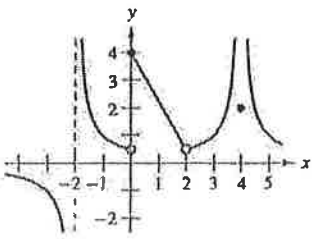


In Exercises 25 and 26, use the graph of the function f to decide whether the value of the given quantity exists. If it does, find it. If not, explain why.

25. (a) $f(1)$
    (b) $\lim_{x \to 1} f(x)$
    (c) $f(4)$
    (d) $\lim_{x \to 4} f(x)$



26. (a) $f(-2)$
    (b) $\lim_{x \to -2} f(x)$
    (c) $f(0)$
    (d) $\lim_{x \to 0} f(x)$
    (e) $f(2)$
    (f) $\lim_{x \to 2} f(x)$
    (g) $f(4)$
    (h) $\lim_{x \to 4} f(x)$



Section 1.3 - Evaluating Limits Analytically:

In Exercises 5–22, find the limit.

5. $\lim_{x \to 2} x^3$

6. $\lim_{x \to -2} x^4$

15. $\lim_{x \to -4} (x + 3)^2$

16. $\lim_{x \to 0} (2x - 1)^3$

17. $\lim_{x \to 2} \frac{1}{x}$

18. $\lim_{x \to -3} \frac{2}{x + 2}$

XIII. Determine all points of intersection

1. parabola $y = x^2 + 3x - 4$ and line $y = 5x + 11$
2. $y = \cos x$ and $y = \sin x$ in the first quadrant

XIV. Solve for x, where x is a real number. Show the work that leads to your solution.

1. $x^2 + 3x - 4 = 14$
2. $\frac{x^4 - 1}{x^3} = 0$
3. $(x - 5)^2 = 9$
4. $2x^2 + 5x = 8$
5. $(x + 3)(x - 3) > 0$
6. $x^2 - 2x - 15 \leq 0$
7. $12x^2 = 3x$
8. $\sin 2x = \sin x, \ 0 \leq x \leq 2\pi$
9. $|x - 3| < 7$
10. $(x + 1)^2(x - 2) + (x + 1)(x - 2)^2 = 0$
11. $27^{2x} = 9^{x-3}$
12. $\log x + \log(x - 3) = 1$

XIV. Graph each function. Give its domain and range.

4. $y = x^3 - 2x^2 - 3x$
5. $y = x^2 - 6x + 1$
6. $y = \frac{x + 4}{x - 1}$
7. $y = \frac{x^2 - 4}{x + 2}$
8. $y = e^x$
10. $y = \sqrt[3]{x}$
11. $y = \ln x$
12. $y = |x + 3| - 2$
14. $y = \begin{cases} x^2 & \text{if } x < 0 \\ x + 2 & \text{if } 0 \leq x \leq 3 \\ 4 & \text{if } x > 3 \end{cases}$

In Exercises 27–30, evaluate the function as indicated. Determine its domain and range.

27. $f(x) = \begin{cases} 2x + 1, & x < 0 \\ 2x + 2, & x \geq 0 \end{cases}$

(a) $f(-1)$ (b) $f(0)$ (c) $f(2)$ (d) $f(t^2 + 1)$

28. $f(x) = \begin{cases} x^2 + 2, & x \leq 1 \\ 2x^2 + 2, & x > 1 \end{cases}$

(a) $f(-2)$ (b) $f(0)$ (c) $f(1)$ (d) $f(s^2 + 2)$

29. $f(x) = \begin{cases} |x| + 1, & x < 1 \\ -x + 1, & x \geq 1 \end{cases}$

(a) $f(-3)$ (b) $f(1)$ (c) $f(3)$ (d) $f(b^2 + 1)$

30. $f(x) = \begin{cases} \sqrt{x + 4}, & x \leq 5 \\ (x - 5)^2, & x > 5 \end{cases}$

(a) $f(-3)$ (b) $f(0)$ (c) $f(5)$ (d) $f(10)$

In Exercises 23–26, find the limits.

23. $f(x) = 5 - x,\ g(x) = x^3$

(a) $\lim_{x \to 1} f(x)$ (b) $\lim_{x \to 4} g(x)$ (c) $\lim_{x \to 1} g(f(x))$

24. $f(x) = x + 7,\ g(x) = x^2$

(a) $\lim_{x \to -3} f(x)$ (b) $\lim_{x \to 4} g(x)$ (c) $\lim_{x \to -3} g(f(x))$

25. $f(x) = 4 - x^2,\ g(x) = \sqrt{x + 1}$

(a) $\lim_{x \to 1} f(x)$ (b) $\lim_{x \to 3} g(x)$ (c) $\lim_{x \to 1} g(f(x))$

26. $f(x) = 2x^2 - 3x + 1,\ g(x) = \sqrt[3]{x + 6}$

(a) $\lim_{x \to 4} f(x)$ (b) $\lim_{x \to 21} g(x)$ (c) $\lim_{x \to 4} g(f(x))$

In Exercises 27–36, find the limit of the trigonometric function.

27. $\lim_{x \to \pi/2} \sin x$

28. $\lim_{x \to \pi} \tan x$

29. $\lim_{x \to 1} \cos \frac{\pi x}{3}$

30. $\lim_{x \to 2} \sin \frac{\pi x}{2}$

31. $\lim_{x \to 0} \sec 2x$

32. $\lim_{x \to \pi} \cos 3x$

33. $\lim_{x \to 5\pi/6} \sin x$

34. $\lim_{x \to 5\pi/3} \cos x$

35. $\lim_{x \to 3} \tan\left(\frac{\pi x}{4}\right)$

36. $\lim_{x \to 7} \sec\left(\frac{\pi x}{6}\right)$